Mathematics as a Laboratory Science

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Abstract

The aim of this paper is to analyze the role of physics laboratory in teaching mathematics. In order to clarify the relation between mathematics and physics, theoretical arguments and existing literature are reviewed. Physics can contribute to mathematics teaching in two aspects: as real life problems describing a physical situation demanding the appropriate mathematical model (before the teaching of the correspondent math concept or algorithm) or as an imitation of the real situation in the context of the laboratory. The latter is investigated in our work. Working in the laboratory, secondary school students used science and engineering practices, such as asking questions and defining problems, developed and used models, and carried out investigations. The intersecting concepts of cause-effect and function were discussed initially at an intuitive level, and in a more abstract one after the laboratory session.

Keywords: mathematics teaching, laboratory, physics, experiment

The role of Physics in Mathematics teaching

Until the 19th century the disciplines of mathematics and physics were not separate; major figures such as Galileo, Kepler, Leibniz, Newton cannot be considered as either physicists or mathematicians, rather they were Natural Philosophers. Poincaré, preoccupied by many aspects of mathematics, physics and philosophy, is often described as the last universalist in mathematics. The real divide between Mathematics and Physics began to open up in the 19th century. Morris Kline (1980) has written about "the disastrous divorce" of the mathematics profession from physics, which began in the latter part of the nineteenth century. He estimated that, by 1980, eighty per cent of active mathematicians were ignorant of science and perfectly happy to remain that way. In the tertiary level, mathematics courses have become increasingly irrelevant to physics, so physics departments offer their own courses in "methods of mathematical physics" at both graduate and undergraduate levels. Nevertheless, Mathematics has always been the language of Physics and Physical experiments. Physical Mathematics is defined as a process of knowledge creation with the intention to develop mathematical models of physical phenomena, and is [motivated] by them, in contrast with mathematical physics, which historically deals with concrete applications of mathematics to physics. According to Dirac (1939, p.3): "Mathematics and physics are becoming ever more closely connected, though their methods remain different. One may describe the situation by saying that the mathematician plays a game in which he himself invents the rules while the physicist plays a game in which the rules are provided by Nature, but as time goes on it becomes increasingly evident that the rules which the mathematician finds interesting are the same as those which Nature has chosen. It is difficult to predict what the result of all this will be. Possibly, the two subjects will ultimately unify, every branch of pure mathematics then having its physical application, its importance in physics being proportional to its interest in mathematics". According to Moore (2014), physical mathematics is sometimes viewed with suspicion by both physicists and mathematicians. On one hand, mathematicians regard it as deficient, for lack of mathematical rigor, on the other hand, its relative lack of reliance on laboratory experiments is viewed - with some justification - as dangerous by many physicists. In the field of education in particular, the role of mathematics in physics teaching and learning, is well-established. It is well-known that when it comes to solving physics problems, mathematics can be of major assistance, as mathematical models are often used to describe physical events in the real world. The opposite, is less evident. Sometimes reflecting on physical principles, or imagined physical set-ups, can lead to the discovery of mathematical truths. In the classroom, the role of physics in mathematics is restricted to some problems introduced only after the students have been taught about the mathematical concept, though physics can be a fertile ground for new mathematical ideas and creative mathematical reasoning. In spite of efforts to restructure mathematics and science curricula, conventional uses of the textbook, and methods of delivery have remained unchanged. Only rarely are mathematics and science in real interdisciplinary contexts combined in textbooks and teaching materials, carving a divide between formal mathematics -an island- and real world experience and applications -the mainland- in a so-called "island problem". This is especially evident in upper secondary education, where decontextualized theories and specialized algebraic techniques are simply memorized by students and discarded from memory after exams. More than a century ago Fehr (1963, p.395) had written that: "no subject can contribute more than physics to the teaching of mathematics[...] to the extent that mathematics consists of abstractions from the physical world and physics makes use of mathematical terms and relations to describe aspects of physical situations, the subjects are identical." This contribution can manifest as either a real life problem to be addressed, described before the teaching of the correspondent math concept or algorithm (a physical situation requiring an applicable mathematical model), or as a simulation of the real situation in the context of laboratory.

In this study we address the latter. The laboratory environment we have adopted was a kind of interactive-engagement lab (Hake, 1997) i.e. inquiry-driven, but the students are guided in their inquiry by carefully designed instructions, technology, and teacher support. The theoretical construct guiding the instruments used (: different kinds of bottles), was variation theory, developed by Marton and his collaborators (2004). According this theory, people discern certain aspects of their environment by experiencing variation. When one aspect of a phenomenon or an event varies, while one or more aspects remain the same, the one that changes will be discerned.

Mathematics and Physics: Different styles of thinking?

Is it possible to achieve a mathematics and physics integration, given some epistemological constrains: differences in terminology, notational systems, and styles of thinking? For example, to students, a graph in mathematics constitutes the representation of a function. In applied mathematics and science, the graph comes to represent a relationship between at least two quantities each with its own variable. Crombie (Crombie, 1981, p. 284; quoted in Hacking, 2002, p. 161) describes six styles -not mutually exclusive- of scientific thinking distinguished by their objects and their methods of reasoning. The first three of these methods are: (a) The simple method of postulation, (b) Experimental measurement, and exploration of more complex observable relations. (c) Hypothetical construction of analogical models. Style (a) refers to the Greek search for first principles, while (b) and (c) correspond to the contemporary distinction between experimenter/empirical [empiricist?] and theoretician. In addition, Hacking (2002) postulates a combination of (b) and (c) in a new style, which he calls "the laboratory style of thinking and doing". It encompasses the creation of a class of phenomena, which provide empirical constraints for hypothetical models, and need to be explained, or accounted for, by suitable models (p.658).

Can a laboratory style of thinking be fostered in the mathematics classroom?

Put in a more provocative manner: Can we say that mathematics is a laboratory science? This is rather a philosophical question beyond the scope of this paper, though near equivalent reformulations could be: Are school-mathematics a laboratory science? Is the physics laboratory the place where physics and mathematics can meet each other?

Can physics and mathematics be a "happy family" in which theory and experiment coming from different directions meet? (1983, p. 159). Given that cognitive processes for understanding math and physics are intimately linked and fundamentally the same (Hestenes, 2010, p.14), we could answer affirmatively to these questions. According to Hacking (2007), there is no incompatibility between the laboratory style of thought and action, and mathematics teaching and learning: "In the case of mathematics, we are familiar on the one hand with a distinction between mathematical and other methods of reasoning, and on the other, with a distinction between the abstract objects of mathematics, and the objects of everyday life."

Furthermore, in mathematics and physics, problems are the source of new knowledge constructed through modeling processes. Knowledge of mathematics and physics (at least until the beginning of university) involves relations between two worlds: theory-model and object-event worlds. During the laboratory work, students are expected to link observed data to either theoretical models, or to the 'real problem' that they are investigating.

The basic problem of mathematical modeling in a laboratory context is that non-abstract constructs (apparatus, instruments…) are, by virtue of being non-abstract, essentially different from abstract mathematical objects. So, in the process of abstraction the content of the mathematical result can never be the same as the content of its interpretation in terms of non-abstract constructs. Therefore, informal personal decisions and social negotiations are required about whether and to what extent the interpretation can be accepted. Measurement, itself be seen as an instance of contradiction between the observational data and the abstract construct. As Hennig (2010, p.25) comments, since mathematical and non abstract constructs do not belong to the same domain of reality, they cannot be identified with each other […] Mathematical modeling can support agreement about the modeled reality, as long as the model and its interpretation can be accepted by everyone involved […] Communication is also required about measurement procedures. The consequence of these statements is that the designed laboratory activities require carefully chosen experiments, in order to lead the students to construct the demanded relationships. A crucial element in the laboratory experiments is the artifice used.

Teaching mathematics in the classroom is totally different than teaching in the physics laboratory. The first has the teacher as the protagonist. The second, the "laboratory style" teaching, place on the scene "A new actor: not a person but a piece of apparatus" (Hacking 2007, p.8). This apparatus, the “artifice” is not merely a mechanical device, but creates new phenomena. In the laboratory, instruments generally start out as objects of investigation before they can be trusted as tools to produce new phenomena, not bound to the original research. Under this line of reasoning several questions concerning the teaching of mathematics arise: How can we trace the way from laboratory apparatus to mathematical objects/abstractions? What work has to be done to make experimental phenomena travel to locations away from where they were constructed? Golinski (2008, p.33) labels this "the problem of construction": “In a sense, this is the same problem that traditional philosophy of science has tackled under the heading, "the problem of induction," with the assumption that the question is one of legitimating the form of argument that moves from a particular instance of a phenomenon to a general law”. Fleck (1979) provided two answers to this question. One concerns the mechanisms of communication (discursive level): “The same process occurs in the course of translation from reports in scientific journals to textbook science. In textbooks, facts are consolidated, simplified, and stripped of reference to the particular circumstances of their origins; they thereby become more certain as knowledge”. The other concerns the transfer of the sustaining culture of thought collectives to new sites “Phenomena are then reproduced outside the laboratory by transferring the conditions prevailing in the "microworlds" to other settings” (p.34). Both questions were subsequently integrated and developed within constructivist studies. Rouse (1987) refers to the processes of “standardization". This, does not means that " scientific knowledge has no universality, but rather that what universality it has is an achievement always rooted in local know-how within the specially constructed laboratory setting" (p. 119). Put in the context of laboratory science, mathematics becomes an investigation of interesting phenomena, and the student enters the shoes of the scientist: observing, recording, manipulating, predicting, conjecturing and testing, and developing theories as explanations for the phenomena. Laboratory mathematics differ from experimental mathematics. “Experimental mathematics” is a name that has been loosely given to a new mode of doing mathematical research where the computer is used as a “laboratory,” and the “data” are the results of mathematical computation. Using this methodology, we can “see” results long before we can rigorously prove them, and in fact, the experimental results may point the direction of formal proofs”(Bailey and Borwein 2009, p.13)

Interdisciplinary teaching: Looking for …concepts

The title of the World Mathematical Yearbook 2000 of the American Mathematical Society was “Frontiers and Perspectives”. In the books’ preface Michael Atiyah pondering over mathematics in the 21st century, concludes that the close symbiotic relationship between mathematics and physics will rise to new heights in the 21st century. To the late Russian mathematician Vladimir Arnold (1998), mathematics constitutes the part of physics where experiments are cheap. He marks the catastrophic repercussions of attempts to separate physics and mathematics, and warns that such attempts result in teaching ugly scholastic pseudo-mathematics to schoolchildren, where the scheme used in physics (observation, model, investigation of the model, conclusion, testing by observations) is replaced by the scheme definition, theorem, and proof. It is often highlighted in science education that the best description of many phenomena and their patterns of interaction is achieved through the language of mathematics, which consequently bridges the scientific meaning we seek to express with the students' verbal language (Osborne 2002). According Fehr (1963, p.395), “secondary school mathematics and physics, scarcely reach the levels of abstractions which make them separate, independent bodies of knowledge. Hence we are concerned here with single descriptions of matter, space, time and motion, so far as these notions may be expressed mathematically and in an intuitive manner. Our point of view is expressed in the word intuition by which we mean that we are concerned in physics with that which we can “look at”, “feel, or “hear, and with the mathematical concepts involved in this kind of physics. We select those physical concepts that are so intuitively simple that we do not need to tech physics in our mathematics classes. We select those physical properties which are explicitly described by the mathematics we intend our students to study”. In order to comprehend and be involved with phenomena from across disciplines, students need to acquire new concepts, new mental tools, which will assist them in deepening their understanding of the disciplinary core ideas and develop a coherent and scientifically based view of the world. These are referred to as “crosscutting concepts” in the literature. “Crosscutting concepts have value because they provide students with connections and intellectual tools that are related across the differing areas of disciplinary content and can enrich their application of practices and their understanding of core ideas”. Some crosscutting concepts are as follows: Patterns, Cause and Effect, Scale-proportion and quantity, system and systems models, structure and function, stability and change. It is necessary for teachers to incorporate a small number of crosscutting concepts into each yearly curriculum, instead of attempting to teach all of them simultaneously. Function is an essential crosscutting concept, which has become one of the fundamental ideas of modern mathematics and other disciplines. According to Selden & Selden (1992), one of the most common uses of functions is modeling the real-world to help organize the physical world is one of the most common uses of functions. Sierpinska (1992) suggests that functions should first appear as an appropriate tool for mathematizing relationships between physical (and other) magnitudes. "The use of the concepts of instantaneous velocity and acceleration of non linear motion in introducing the first ideas of continuity and the derivative in the calculus is well known. It must be remembered that the purpose is not to teach physics, but to use physics as the first step towards the learning of differential and integral calculus".

Putting the theory in practice: Teaching the "rate of change" in the laboratory

Our team consisting of 5 school-teachers, 2 counselors and one researcher, developed a "laboratory course" following the Lesson Study model. The object of our course was the concept of "rate of change". It is a crosscutting concept appearing in the curricula of other disciplines (such as physics, chemistry, biology), very often used in modeling of real life situations, and crucial for the understanding of functions (Carlson, 1998; Carlson, Jacobs, Coe, Larsen & Hsu, 2002). The phenomenon of filling bottles in the physics laboratory was used as an initial activity. Students had to imagine bottles of different shapes, being filled with water at a constant rate (Carlson et al, 2002). The two covariating quantities were the volume and the level of the water in the bottle. Students had to conduct precise measurements, organizing them in a table of values and plotting them on a Cartesian system. Research has shown that, since students primarily study ratios and linear functions, they tend to utilise linear relationships even when they are not applicable. This phenomenon is called "predominance of linearity" (Dooren et al., 2008, p. 311). If this tendency is combined with the drawing of an incomplete graph, it can severly affect the gauging of the situation. And that is because students do not regard it as a "approximate image" of the relationships, rather than as a faithful depictionof the situation. The concept of experimental error does not exist, a fact which deteriorates the modeling process.Students are used to working with "false-real" problems, where measurements are riven and oftentimes interger, or with precise measurements on a computer. Through communication among team members, the measurement results were doubted. The passage from (dubious) empirical data to the (even intuitive) perception of the rate of change of the height relative to the shape of the container, allows us to formulate the hypothesis about the positive role of the experiment in approaching certain abstract mathematical concept. Granted, further research is required.

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